INHOMOGENEOUS MARKOV CHAIN APPROACH TO PROBABILISTIC SWARM GUIDANCE ALGORITHMS

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ABSTRACT

Small satellites are well suited for formation flying missions, where multiple satellites operate together in a cluster or predefined geometry to accomplish the task of a single conventional large satellite. In comparison with traditional large satellites, small satellites are modular in nature and offer low development cost by enabling rapid manufacturing using commercial-off-the-shelf components. Flight of swarms of hundreds to thousands of femtosatellites (100-gram-class satellites) for Synthetic Aperture applications has been discussed in [1].

A probabilistic guidance approach, which provides a method for each agent to determine its own trajectory such that the overall swarm converges to a desired distribution, while maintaining no communication between agents, is discussed in [2]. Instead of allocating agent positions ahead of time, probabilistic guidance is based on designing a homogeneous Markov chain, such that the steady-state distribution corresponds to the desired swarm density. Although each agent propagates its position in a statistically independent manner, the swarm asymptotically converges to the desired steady-state distribution associated with the homogeneous Markov chain and also automatically repairs any damage. Similar study on self-organization of swarms using homogeneous Markov chains has been done in [3]. The desired Markov matrices, to guide individual swarm agents in a completely decentralized fashion, are synthesized using the Metropolis-Hastings algorithm [4].

The limitations of probabilistic guidance using homogeneous Markov chains are (i) the agents are not allowed to settle down even after the swarm has reached the desired steady-state distribution resulting in significant fuel loss, and (ii) only asymptotic guarantees of convergence are provided. This paper develops probabilistic swarm guidance algorithms using inhomogeneous Markov chains to address these limitations. In order to achieve these objectives, it is necessary that each agent senses the agents in its surroundings and also communicate with its neighboring agents.

The motion of agents in the swarm can be viewed as analogous to the random motion of molecules in an ideal gas. Just as the temperature dictates the motion of molecules in a gas; this paper uses the Kullback–Leibler (KL) divergence between the current swarm distribution and the desired steady-state distribution, to dictate the motion of agents in the swarm. The KL divergence is a non-symmetric measure of
the difference between two probability distributions [5]. Each agent senses the agents in its surroundings and makes a localized guess of the current swarm distribution. Using the Bayesian Consensus Filtering algorithm, the agents reach a consensus regarding the global current swarm distribution [6].

The next step involves synthesizing a series of inhomogeneous Markov matrices, such that as the KL divergence decreases, the Markov matrices tend towards an identity matrix. The conditions for existence of such series of inhomogeneous Markov matrices, called ‘hardening-position scheme’, is discussed in the [7]. In essence, when the KL divergence between the current swarm distribution and the desired steady-state distribution is large, each agent propagates its position in a statistically-independent manner, and the swarm tends toward the desired steady-state distribution. When this KL divergence is small, the Markov matrix tends towards an identity matrix and each agent holds its own position.

As shown in Figure 1, this inhomogeneous Markov chain solves the first limitation by allowing the agents to settle down when the swarm has reached the desired steady-state distribution. Moreover, global exponential convergence can be guaranteed since the KL divergence essentially provides a feedback of the current swarm distribution. In addition, the self-repair property of the swarm is maintained, as any damage will change the KL divergence, which will trigger the agents back into motion. We envisage that fuel efficient probabilistic swarm guidance algorithms using inhomogeneous Markov chains can be effectively used in formation flying missions of swarms of femtosatellites.

References: