Abstract: The in-flight performance validation of the experimental autonomous formation keeping system embarked by the German TanDEM-X formation has been performed during a 12-day-long closed-loop campaign conducted in June 2012. Relative control performance better than 10 m was achieved, demonstrating that a significant gain of performance can be achieved when the control of the formation is done autonomously on-board instead of on-ground. Furthermore, the formation keeping system was shown to be operationally robust, easy to operate and fully predictable, i.e. fully suited for routine mission operations. This campaign concludes successfully a series of validation activities, opening new doors to future innovative scientific TanDEM-X experiments for which enhanced formation control is required.

Keywords: TanDEM-X, TAFF, autonomy, formation keeping, Along-Track Interferometry.

1. Introduction

The TerraSAR-X mission (TSX, launched on June 15th 2007, operated in 505 km, sun-synchronous, low Earth orbit) provides high-resolution Synthetic Aperture Radar (SAR) data to both science and commercial users. An almost identical satellite, TanDEM-X (TDX), was launched on June 21st 2010 in order to form the first configurable SAR interferometer employing formation flying with TSX. The main objective of the common TanDEM-X (TerraSAR-X add-on for Digital Elevation Measurement) mission is to generate a global digital elevation model (DEM) with unprecedented accuracy as the basis for a wide range of scientific research as well as for commercial DEM production. In order to collect sufficient measurements for a global DEM, three years of formation flying are foreseen with changing across-track baselines ranging from 150 m to few kilometers [1].

After the completion of the DEM acquisition, the mission will offer a unique opportunity to investigate and demonstrate innovative SAR techniques such as Along-Track Interferometry (ATI) e.g. for the measurement of ocean currents. Such a SAR application is much more demanding in terms of formation control performance. Contrary to the coarse 200 m along-track relative control accuracy required for routine global DEM acquisition, the along-track separation desired for ATI oceanography is only 50 m ± 10 m [2]. This is quite challenging for a ground-in-the-loop formation control with typically 30 m rms along-track control accuracy [3]. Fortunately, the TanDEM-X formation is also equipped with an onboard autonomous formation keeping system, called TanDEM-X Autonomous Formation Flying (TAFF), able to fulfill such a control requirement. TAFF is an embedded system able to take over on demand the in-plane
Two closed-loop campaigns have been conducted to validate the functional behavior of TAFF and assess the formation control performance. The first closed-loop test campaign [4] was conducted in March 2011. As described in [4], this three-day-long test campaign could demonstrate the ability of TAFF to maintain correctly the formation as well as the proper implementation of numerous safety mechanisms to ensure that the autonomous formation keeping system would never cause any kind of disturbance to the mission objectives, whatever happens. However, the autonomous control of the formation was partly inhibited because of unexpected degradation of the navigation performance of the GPS receivers [4]. After identification and fix of the performance problems affecting the GPS receivers, a second longer (12 day-long) campaign has been conducted one year later. Its objective was not only to assess the formation keeping performance but also to investigate the ability of the formation keeping system to quickly reconfigure the formation baseline in view of its potential utilization to support future ATI experiments. After a brief description of TAFF, the paper presents the key results collected during this campaign and the lessons learned so far.

2. The TanDEM-X Autonomous Formation Flying System

Being implemented as experimental add-on in a highly sensitive mission, TAFF undergoes severe operational constraints. In particular the system is not allowed to use any thruster during the acquisition of SAR images. Furthermore, a deterministic control scheme is mandatory to facilitate the planning of operations and to ease the monitoring of the autonomous system. Finally a low computational load is required to cope with limited onboard resources. This led to the adoption of streamlined navigation and control algorithms focused on operational robustness and simplicity during the system design and implementation. TAFF is a standalone (relative) Guidance, Navigation and Control (GNC) system implemented as part of the Attitude and Orbit Control System (AOCS) of TDX. The navigation and control algorithms are based on a special parameterization of the relative motion using a set of six relative orbit elements defined as follows [5]:

\[
\Delta \alpha = \begin{pmatrix}
\Delta a \\
\frac{\Delta a}{a_1} e_x \\
\frac{\Delta a}{a_1} e_y \\
\frac{\Delta a}{a_1} i_x \\
\frac{\Delta a}{a_1} i_y \\
\frac{\Delta a}{a_1} u 
\end{pmatrix} = \begin{pmatrix}
a_1 - a_2 \\
\frac{a_1}{a_2} (e_2 \cos(\omega_2) - e_1 \cos(\omega_1)) \\
\frac{a_1}{a_2} (e_2 \sin(\omega_2) - e_1 \sin(\omega_1)) \\
\frac{a_1}{a_2} (i_2 - i_1) \\
\frac{a_1}{a_2} (\Omega_2 - \Omega_1) \sin(i_1) \\
\frac{a_1}{a_2} (u_2 - u_1)
\end{pmatrix}
\] (1)

where \(a, e, i, \omega, \Omega, u\) denote respectively the semi-major axis, eccentricity, inclination, argument of perigee, right ascension of the ascending node and mean argument of latitude of the spacecraft identified by the subscript 1 and 2. In the state vector the non-dimensional orbit elements have been multiplied by the semi-major axis of the first spacecraft to obtain consistent units among all the components of the vector. Since the spacecraft are flying in close formation, the quadratic differences of orbit elements are neglected in the linearization process. As a consequence the subscripts of the absolute orbit elements are removed in the sequel. The state defined by Eq. (1) describes uniquely and unambiguously the formation geometry.
and can be used to express in a convenient way the solution of the Hill-Clohessy-Wiltshire equations of motion [5]:

\[
\begin{align*}
\Delta r_R &= \Delta a - a \Delta e_x \cdot \cos (u) - a \Delta e_y \cdot \sin (u) \\
\Delta r_T &= a \Delta u + a \Delta i_y \cdot \cot (i) - \frac{3}{2} a \Delta a \cdot (u - u_0) - 2 \cdot a \Delta e_y \cdot \cos (u) + 2 \cdot a \Delta e_x \cdot \sin (u) \\
\Delta r_N &= -a \Delta i_y \cdot \cos (u) + a \Delta i_x \cdot \sin (u)
\end{align*}
\]

(2)

Here, \( \Delta r_R, \Delta r_T \) and \( \Delta r_N \) are the radial, along-track and normal components of the relative position expressed in a local co-moving Radial-Tangential-Normal (RTN) frame originating in the TDX center of mass. The main advantage of this parameterization is to allow a quick insight in the geometry of the formation and to provide at the same time a simple criterion to assess the risk of collision [5]. The relative navigation module uses as measurements the Earth-fixed positions coming from the MosaicGNSS GPS receivers onboard TDX and TSX and implements a Kalman filter to estimate the six relative orbit elements describing the formation as defined in Eq. (1). The GPS navigation solution of TSX is transmitted in real-time to the TDX spacecraft via an inter-satellite link. The filtering is done using a simple dynamical model which considers only the perturbation due to the Earth’s equatorial bulge \( J_2 \), resulting in a secular rotation of the so-called dimensioned relative eccentricity vector \( a \Delta e \). The constant angular drift of \( a \Delta e \) is called in the sequel \( \dot{\phi} \), \( \phi \) denoting the phase of the vector \( \Delta e \). According to this model, the relative semi-major axis \( \Delta a \) and the dimensioned relative inclination vector \( a \Delta i \) are constant over time while the time evolution of relative mean argument of latitude \( a \Delta u \) is proportional to \( \Delta a \). Mathematically, this dynamical model can be written as:

\[
\Delta \dot{\alpha} = \begin{pmatrix} 0 & -\dot{\phi} a \Delta e_x & \dot{\phi} a \Delta e_y & 0 & 0 & -\frac{3}{2} n \Delta a \end{pmatrix}^T
\]

(3)

where \( n \) denote the spacecraft mean motion. This filter design does certainly not offer ultimate relative navigation performance but guarantees a very limited usage of onboard resources, because it does not require any numerical integration nor any processing of raw GPS measurements. The filter provides in addition a continuous real-time onboard relative navigation even in the presence of GPS data gaps.

The TDX spacecraft is equipped with 1 N hydrazine thrusters for absolute orbit control and 40 mN cold-gas thrusters poiting in flight and anti-flight directions for formation maintenance. For simplicity, it has been decided that TAFF performs only in-plane formation maintenance to avoid any rotation of the spacecraft necessary to align the cold-gas thrusters in the out-of-plane direction. The formation keeping is based on an analytical solution of the relative control problem. The onboard controller makes use of pairs of maneuvers separated by half an orbit to keep the relative orbit elements \( \Delta a, a \Delta e \) and \( a \Delta u \) close to their nominal values \( (a \Delta i \) is not controlled because it drives the out-of-plane motion, cf. Eq. 2). The pairs of maneuvers are executed on a configurable regular basis, typically every 5 orbits, to ensure a deterministic control behavior and to ease the mission planning activities.

More detailed explanations about the underlying relative navigation and control algorithms can be found in [6]. In addition TAFF allows for independent spacecraft maneuvering from ground
(typically when using hydrazine maneuvers). In this case TAFF includes the maneuvers in the relative navigation solution during the filter time update and disables its controller to avoid any conflict. The a-priori information about the maneuvers commanded on ground is sent per telecommand to TAFF shortly before their execution.

3. Flight Results

3.1. Overview

Overall the second closed-loop campaign was successful. TAFF has been set in closed-loop mode on June 18th at 9:00 UTC and has been controlling the formation until June 30th 00:00 UTC. During this time interval, 88 maneuvers were executed autonomously to maintain and reconfigure the formation. The formation to be maintained was defined as follows (aΔ i is not controlled by TAFF but has been mentioned for completeness):

Table 1. Nominal formation configurations during the closed-loop campaign [m].

<table>
<thead>
<tr>
<th>Date (June 2012)</th>
<th>Δa</th>
<th>aΔe_x</th>
<th>aΔe_y</th>
<th>aΔi_x</th>
<th>aΔi_y</th>
<th>aΔu</th>
</tr>
</thead>
<tbody>
<tr>
<td>18th 10:00 - 24th 18:00</td>
<td>0</td>
<td>68</td>
<td>387</td>
<td>0</td>
<td>-449</td>
<td>-59</td>
</tr>
<tr>
<td>24th 18:00 - 25th 20:00</td>
<td>0</td>
<td>67</td>
<td>382</td>
<td>0</td>
<td>-445</td>
<td>-88</td>
</tr>
<tr>
<td>25th 20:00 - 26th 20:00</td>
<td>0</td>
<td>67</td>
<td>381</td>
<td>0</td>
<td>-445</td>
<td>2</td>
</tr>
<tr>
<td>25th 20:00 - 30th 00:00</td>
<td>0</td>
<td>67</td>
<td>380</td>
<td>0</td>
<td>-444</td>
<td>-58</td>
</tr>
</tbody>
</table>

According to Table 1, several baselines (i.e. along-track separations) have been assigned in order to reproduce a typical ATI experiment campaign. It is believed that ATI data acquisition will require that a desired along-track separation is achieved over a certain location on the Earth, and that several locations will be visited during a short period of time. As a consequence, the goal at the end of the campaign was to change the formation baseline on a daily basis and observe how fast can be done the baseline reconfiguration and what relative control performance can be achieved. Table 1 shows also that the nominal formation configuration is not constant during the campaign. This is due to the fact that a passive formation reconfiguration was in addition conducted during the closed-loop campaign, during which the nominal size of Δe and Δi was naturally decreasing. In order to cope with this constraint, TAFF was updated daily with new nominal formation configurations (the intermediate formation configurations are not written in Table 1 for clarity).

Fig. 1 depicts the relative orbit elements controlled by TAFF during the campaign. For simplicity only the phase φ of the relative eccentricity vector Δe is depicted because the length of Δe is not altered by J₂. The nominal formation configuration, which has to be maintained by the formation keeping system, is indicated by the means of a green line. The executed maneuvers are depicted by dashed vertical lines. The blue lines represent the cold gas maneuvers autonomously executed by TAFF whereas the red lines stand for the hydrazine maneuvers commanded independently from the ground segment. The hydrazine maneuvers are absolute orbit control maneuvers which are executed simultaneously by TDX and TSX. In principle, the maneuvers are computed to correct the semi-major axes and eccentricities of the absolute
orbits while keeping the formation unaltered. Practically, maneuver execution errors introduce a small differential velocity increment which needs to be compensated.

![Graphs showing relative semi-major axis, relative argument of perigee, and relative mean argument of latitude controlled during the closed-loop campaign.](image)

**Figure 1.** Relative semi-major axis (top), relative argument of perigee (middle) and relative mean argument of latitude (bottom) controlled during the closed-loop campaign

### 3.2. Navigation Performance

Accurate and reliable navigation solutions from the GPS receivers onboard TDX and TSX have undoubtedly contributed to the success of the second closed-loop campaign. Contrary to the first campaign conducted in March 2011, during which the filtered navigation solutions of the MosaicGNSS receivers exhibited large error peaks up to 80 m [4], the second campaign was free of any anomaly due to the GPS receivers. Fig. 2 depicts the errors of the measured Earth-fixed relative position (i.e. the simple difference of the two independent GPS navigation solutions) provided as input to TAFF during the campaign. In the sequel the reference for performance assessment is based on GPS-based precise orbit determination performed routinely at the German Space Operations Center [7]. It can be observed that the relative position error is smaller than 5 m rms, despite some sporadical error peaks up to 30 m. Nevertheless the
onboard navigation filter is able to absorb such error peaks without any notable degradation of the state estimate.

![Error of the GPS-based measured relative position (June 18\textsuperscript{th}-30\textsuperscript{th}, 2012)](image)

**Figure 2. Error of the GPS-based measured relative position (June 18\textsuperscript{th}-30\textsuperscript{th}, 2012)**

As summarized in Table 2, the components of the state vector described in Section 2 are accurately estimated (at the meter level). Despite the simplicity of the filter design, outstanding filtering results are achieved for the estimate of the relative semi-major axis $\Delta a$, affected by an error of only 15 cm. This is of special relevance for accurate along-track relative control, because $\Delta a$ drives directly the time-variation of the along-track separation (cf. Eq. 3).

<table>
<thead>
<tr>
<th>Source</th>
<th>Radial</th>
<th>Tangential</th>
<th>Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Differential GPS navigation</td>
<td>-0.95±3.22</td>
<td>+0.14±2.03</td>
<td>-0.30±1.13</td>
</tr>
<tr>
<td>Onboard relative navigation</td>
<td>-0.01±0.43</td>
<td>+0.48±1.39</td>
<td>-0.05±0.30</td>
</tr>
</tbody>
</table>

Table 2. Error of the onboard state estimate [m] (June 18\textsuperscript{th}-30\textsuperscript{th}, 2012)

In order to ease the comparison with the GPS navigation data processed by TAFF, the relative orbit elements can be mapped using Eq. 2 into the estimated relative position expressed in the RTN frame (cf. Section 2). According to Table 3 the relative navigation performance is at the sub-meter level for the radial and cross-track component and amounts to about 2 m for the along-track component. Here again, it can be observed that the best filtering performance concerns the radial component, for which an improvement of one order of magnitude is achieved. On the contrary, the filter does almost not improve the knowledge of the along-track component.

Table 3. Relative position error [m] (June 18\textsuperscript{th}-30\textsuperscript{th}, 2012)
3.3. Control Performance

Fig. 3 shows the formation keeping errors observed during the closed-loop campaign. Since TAFF performs only in-plane relative control, only the radial and along-track components are depicted. The formation control errors are computed by comparing the relative position of the formation with the relative position obtained by mapping (cf. Eq. 2) the nominal relative orbit elements described in Table 1 into a relative position. The influence of ground-commanded hydrazine maneuvers is clearly observable: when a maneuver is going to be executed from ground, TAFF stops autonomously all formation keeping activities. A maneuver exclusion windows of several hours has been configured which prevents any conflicting maneuvers. During this time, the formation is not controlled anymore and the control errors increase rapidly. The sudden discontinuities which appear three times from June 24th to the end of the campaign correspond to the arrival of new telecommands for different baseline reconfiguration. It can be observed that TAFF needs roughly two control periods to converge the new baseline, the control period being defined as the time interval between two pairs of maneuvers.

![Figure 3. In-plane control error (June 18th-30th, 2012)](image)

Different control periods have been investigated during this test campaign, explaining why the time distance between two pairs of maneuvers is decreasing progressively. The purpose of
the investigation was to find out to which extend small control periods can be used reliably to control the formation. Table 4 summarizes the settings adopted throughout the campaign.

Table 4. Control periods tested during the campaign

<table>
<thead>
<tr>
<th>Time interval</th>
<th>Control period [in orbits]</th>
</tr>
</thead>
<tbody>
<tr>
<td>18th 10:00 - 21st 15:00</td>
<td>5</td>
</tr>
<tr>
<td>21st 15:00 - 27th 10:00</td>
<td>4</td>
</tr>
<tr>
<td>27th 10:00 - 30th 00:00</td>
<td>3</td>
</tr>
</tbody>
</table>

Small control periods improve the control performance but might not be reliable, because the corrections computed by the controller become very sensitive to the noise of the navigation solution. This issue is of special relevance for TAFF, because TAFF has a built-in mechanism which prevents the execution of maneuvers outside a predefined tolerance window. Some simple considerations can help better understanding the problem. The greatest natural perturbation acting on the relative eccentricity vector which has to be counteracted by the controller is due to $J_2$, which causes a secular drift $\delta \Delta e$ of the relative eccentricity vector over time. The drift rate amounts to [6]:

$$
\dot{\phi} = \frac{3}{4} n J_2 \left( \frac{R_\oplus}{a(1-e^2)} \right) (5 \cos(i) - 1)
$$

where $R_\oplus$ denotes the Earth’s radius. Over one orbit the relative perigee angle has drifted by the value $\phi$

$$
\delta \phi = \frac{2\pi}{n} \dot{\phi}
$$

For very small control periods, e.g. every 2 orbits, the angular drift amounts to only $\delta \phi \approx 0.46^\circ$ which translates into an error $a\delta \Delta e \approx (\delta \phi \cdot \Delta e_y, -\delta \phi \cdot \Delta e_x)^T$ affecting the nominal relative eccentricity vector. For a formation defined by a nominal dimensioned relative eccentricity vector $a\Delta e = \begin{pmatrix} 70, & 390 \end{pmatrix}^T$ m, the error which needs to be counteracted is only $a\delta \Delta e \approx \begin{pmatrix} -3.1, & 0.6 \end{pmatrix}^T$ m. The problem comes from the fact that the location of the maneuver $u_M$ is computed onboard and depends on the measured desired correction $\delta \Delta e = \Delta e_{desired} - \Delta e$ to be applied [6]

$$
u_M = \arctan \frac{\delta \Delta e_y}{\delta \Delta e_x}
$$

According to Table [2], the navigation errors amount to a large part of $a\delta \Delta e$, which translates into angular error up to several degrees affecting the location of maneuvers. For this reason, a stepwise validation has been adopted, starting for a pretty large control period (a pair of maneuvers is executed every 5 orbits) and reducing progressively the control period until 3 orbits. Table [5] summarizes the relative control performance obtained with different control periods.
In order to have a fair evaluation of performance, the time intervals for the performance assessment have been selected in such a way to avoid the presence of hydrazine maneuvers (because the controller is disabled in this case) and to avoid any reconfiguration (because the controller needs some time to converge).

According to [6], the size of each maneuver depends directly on the correction $\Delta e$:

$$\delta v = \pm \frac{an \| \delta \Delta e \|}{4} \quad (7)$$

so that the theoretical necessary delta-v to compensate the drift of the formation due to $J_2$ amounts per orbit to:

$$\Delta v \approx 2\pi a \cdot |\dot{\phi}| \cdot \| \Delta e \| \approx 0.867 \text{mm/s}. \quad (8)$$

Reducing too much the control period has also the drawback of increasing the propellant consumption, because the formation corrections computed on-board are more affected by navigation noise. From the last column of Table 5, which lists the measured average velocity increment required per orbit by TAFF for each control period, it can be observed that is effect is negligible for the control periods used during the campaign.

Table 5. Radial (R) and along-Track(T) relative control performance

<table>
<thead>
<tr>
<th>Selected time interval (June 2012)</th>
<th>Control period [orbits]</th>
<th>Number of orbits [-]</th>
<th>Relative control error [m] (RMS/max)</th>
<th>delta-v/orbit [mm/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>19th 23:30 - 21st 15:00</td>
<td>5</td>
<td>25</td>
<td>R: 2.0/5.4 T: 8.0/25.5</td>
<td>0.870</td>
</tr>
<tr>
<td>22nd 00:00 - 24th 09:00</td>
<td>4</td>
<td>36</td>
<td>R: 1.7/4.3 T: 5.8/17.4</td>
<td>0.868</td>
</tr>
<tr>
<td>28th 10:00 - 30th 00:00</td>
<td>3</td>
<td>24</td>
<td>R: 1.3/4.7 T: 3.5/13.4</td>
<td>0.856</td>
</tr>
</tbody>
</table>

3.4. System Predictability

The operational aspects are of special relevance for a mission like TanDEM-X. In particular, mission planning activities often require to foresee approximately when the maneuvers will be executed to avoid any conflict with SAR data takes. TAFF demonstrates that onboard autonomy does not necessary imply a lack of comprehension of what is currently happening on the satellite. The TAFF algorithms have been tuned to ensure maximum predictability. This means that TAFF is able to react immediately in case of anomaly (which is the great added value of onboard autonomy) but also that in nominal situations it is possible to plan several days in advance when a maneuver will be executed. Starting from the precise knowledge of the formation on June 18th (retrieved via a GPS-based precise orbit determination) and knowing that hydrazine maneuvers will be executed on June 19th at 11:43 and June 28th at 00:31, it is possible to predict the execution time of all the maneuvers performed by TAFF with a good precision. A simple tool which propagates accurately the orbit of the formation (using a 30x30 gravity field and a model of the atmospheric drag) and reproduces the simple logics of the state machine implemented in TAFF can do it easily. Fig. 4 shows for example
the error of the prediction of the execution time for the 88 maneuvers executed by TAFF during the campaign. It can be seen that a prediction accuracy better than 15 min can be achieved over two weeks. The remaining error affecting the prediction is due to the mismodelling affecting the propagation over two weeks (secular growth) and the fact that the location of maneuver is very sensitive to the relative navigation errors (cf. Section 3.3), which has an impact on the prediction of the maneuver execution time (the formation covers roughly 4° in one minute).

Figure 4. Prediction error of the execution time of 88 maneuvers done by TAFF

3.5. Operational Robustness

No anomaly was detected during the second closed-loop campaign, indicating that the system is pretty robust. Many internal safety mechanisms are implemented on-board to deactivate the controller as soon as the smallest doubt exists about the validity of the relative navigation and to prevent the execution of maneuvers during SAR data takes. These aspects could be deeply investigated during the first closed-loop campaign and are documented in [4].

4. Conclusion

TAFF is the first onboard autonomous formation keeping system ever employed on a high-cost scientific formation flying mission with routine data acquisition. As such, it has to face natural fears and reluctance to rely on onboard autonomy for critical activities like formation maintenance. TAFF aims at making evolving the minds by proving that a proper design of the formation (passively safe) as well as a smart implementation of the onboard navigation software (robust navigation and control, internal safety mechanisms) can guarantee simple, accurate and safe formation keeping.

The gain of control performance obtained by enforcing onboard autonomy is evident: smaller reaction times and smaller control periods lead to enhanced control performance. The closed-loop campaign conducted in June 2012 shows that relative control accuracy better than 10 m can be achieved. Furthermore, TAFF could demonstrate during the closed-loop tests a good operational robustness and simplicity of use. Despite all these advantages, TAFF has never been used routinely for DEM acquisition during the primary mission phase. The main reason is simply because it was not needed: the control performance obtained in a ground in-the-loop scheme is fully sufficient and the formation keeping activities are also automatized on
ground. Both approaches require on ground more or less the same mission operation effort and flight-dynamics support.

Nevertheless, the moment of glory of TAFF might come soon. After the completion of the nominal mission phase, the TanDEM-X mission will serve as platform for new SAR applications requiring frequent baseline reconfigurations and more accurate formation control performance which can hardly be obtained with a ground-in-the-loop scheme. TAFF will then become an essential tool to achieve easily these objectives.

5. Acknowledgement

The TanDEM-X project is partly funded by the German Federal Ministry for Economics and Technology (Förderkennzeichen 50 EE 0601) and is realized in a public-private partnership (PPP) between German Aerospace Center (DLR) and Astrium GmbH.

6. References


