Periodic Relative Orbits of Two Spacecraft Subject to Differential Gravity and Coulomb Forces

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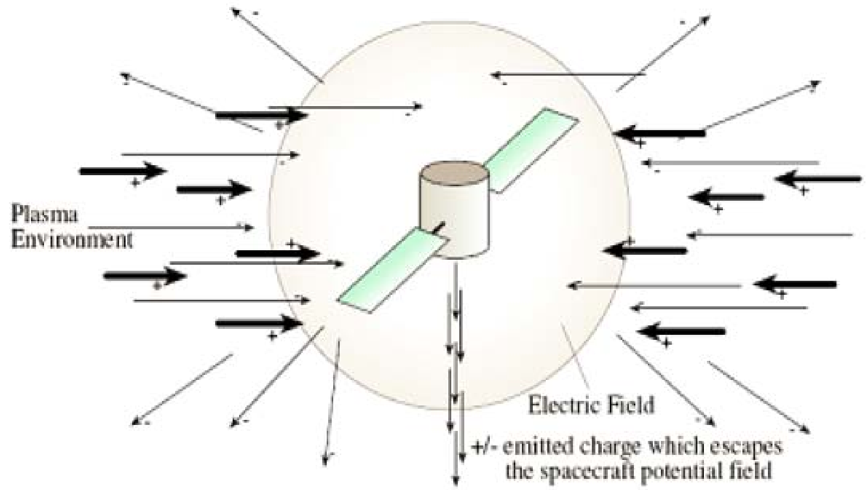

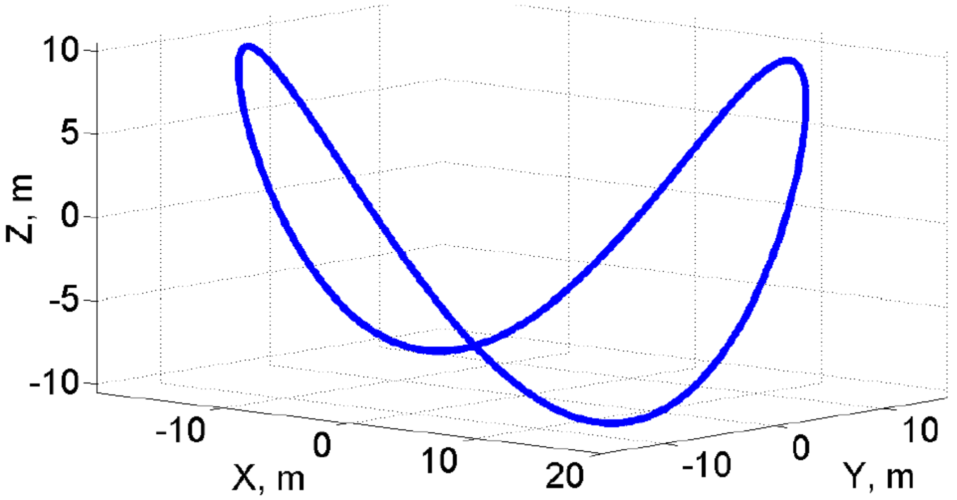

Coulomb Formation Motivation

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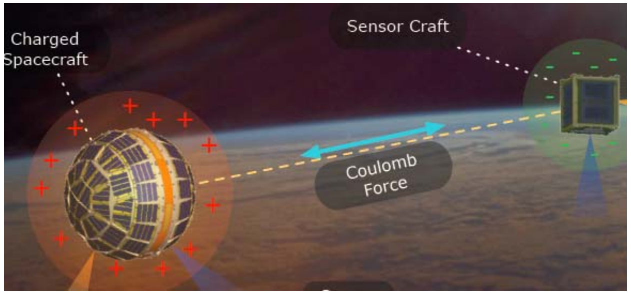
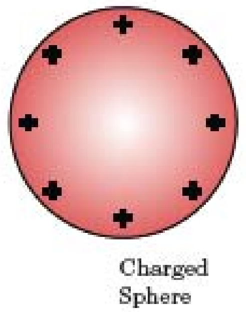
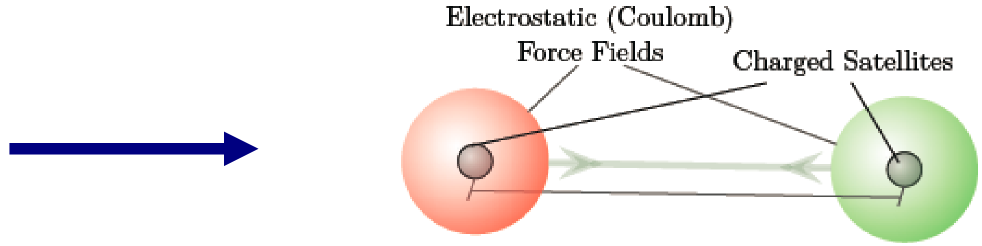
Coulomb Formation: Close-flying charged craft using electrostatic forces

- Very efficient (ISP~10^{13} s)
- Forces have limited reachability
- Forces introduce NL coupling
- Forces complex to model in the plasma







$$F = k_c \frac{q_1 q_2}{r^2}$$

Outline

I. Approximate s/c charging and Coulomb force model

II. Dynamics and dynamical system properties

III. Derivation of periodic Coulomb formation solutions

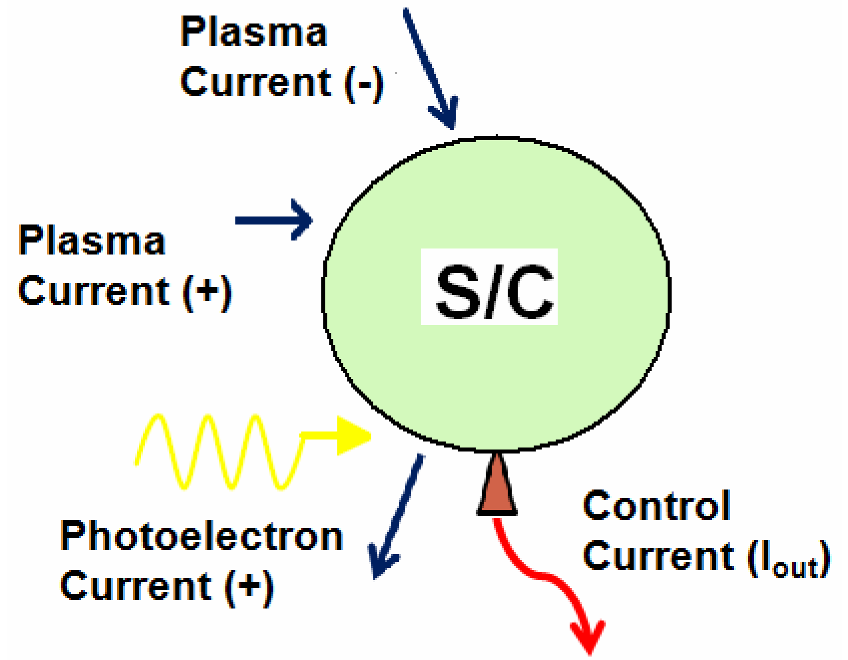
IV. Simulated periodic Coulomb formation solutions

V. Perturbed Coulomb formation motions

VI. Conclusions and future research

Spacecraft Charging

- Spacecraft naturally assume non-zero potential (charge)
- Artificially altering potential has been demonstrated
- Spherical s/c model adopted
  - Somewhat an abstraction
  - Enables 1st order calculations and assumptions

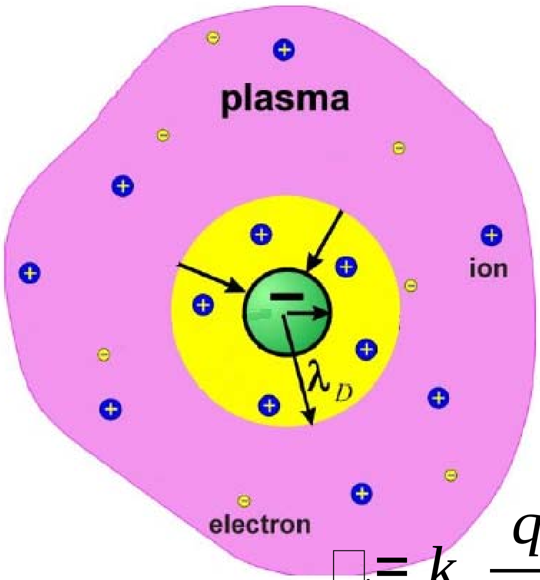


Electrostatic Model

- Spacecraft are not point charges in a vacuum...
  - Adopt approximate and analytical Coulomb force model
  - But still account for plasma shielding effect
  - Assume formations near GEO

Coulomb Force: Point Charges in Vacuum

$$F_{12} = k_c \frac{q_1 q_2}{r_{12}^2}$$



: Finite Bodies with sma Shielding

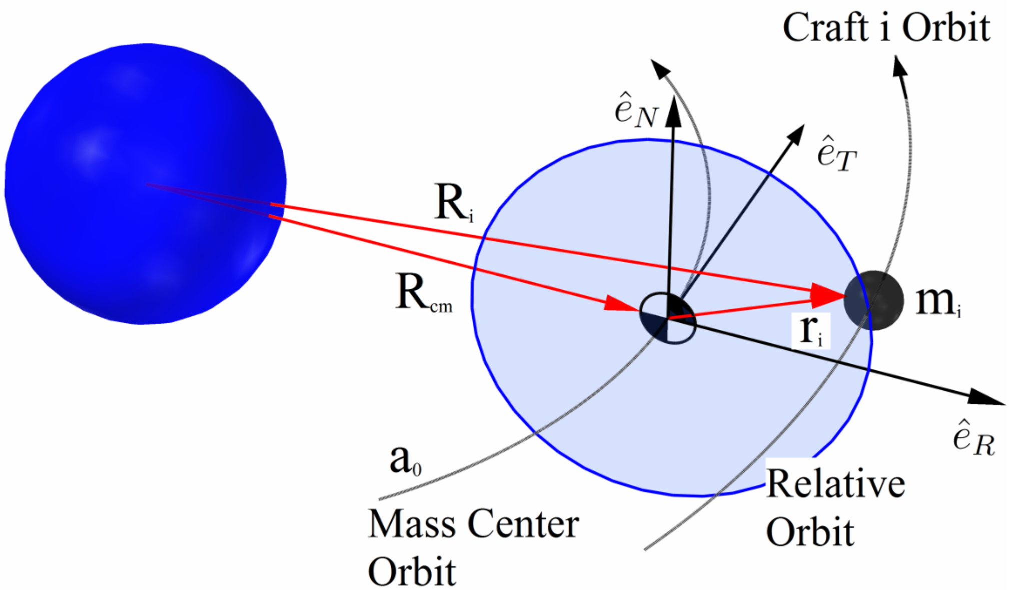
$$\frac{{}_1 q_2 (1 + r_{12}/\lambda_d)}{r_{12}^2 \exp[r_{12}/\lambda_d]}$$

$$\square_i = k_c \frac{q_i}{R_{sc}}$$

Dynamical Model

- Hill Frame Model
- Clohessy-Wiltshire relative motion EOM

$$\sum_i m_i \mathbf{r}_i = 0$$



- With Debye-Hückel Coulomb force model:

$$\ddot{\mathbf{r}}_i = \begin{bmatrix} 2\omega\dot{y}_i + 3\omega^2 x_i \\ -2\omega\dot{x}_i \\ -\omega^2 z_i \end{bmatrix} + \left[\frac{k_c\ q_i}{m_i}\sum_{j\neq i}\frac{q_j\ e^{-r_{ij}/\lambda_d}}{r_{ij}^3}\left(1+\frac{r_{ij}}{\lambda_d}\right)\mathbf{r_{ij}}\right]$$

Reduced Dynamics

- Dynamics normalized, one craft removed
  - s/c 2 motion is always dependent, via CM constraint
  - Scaled charge products introduce time transformation

$$Q_{12} = q_1 q_2$$

$$\tilde{Q}_{12} = \frac{k_c\, Q_{12}}{\omega^2} \qquad \mathrm{d}\tau = \omega \mathrm{d}t \qquad (\zeta)' = \frac{\mathrm{d}\zeta}{\mathrm{d}\tau} = \frac{1}{\omega}\frac{\mathrm{d}\zeta}{\mathrm{d}t}$$

- Scalar constant of motion is shown to exist
  - Holds for N-craft Hill frame systems with internal forcing
  - For two-craft formation, the motion constant is:

$$x(\tau)y''(\tau) - y(\tau)x''(\tau) = 0$$

Dynamical System Analysis

- Seek periodic solutions to vector ODE $\mathbf{X}' = \mathbf{F}\left(\mathbf{X},\ \mathbf{u},\ \tau\right)$
  - Dynamics linearized about such solutions

$$\delta\mathbf{X}'(\tau) = \left(\frac{\partial\mathbf{F}}{\partial\mathbf{X}}\right)\Bigg|_{(\mathbf{X}^*,\mathbf{u}^*)} \delta\mathbf{X}(\tau) \ = \ \mathbf{A}(\tau)\ \delta\mathbf{X}(\tau)$$

$$\delta\mathbf{X}(\tau) = \mathbf{\Phi}(\tau,0)\ \delta\mathbf{X}(0)$$

$$\mathbf{\Phi}'(\tau,0) = \mathbf{A}(\tau)\ \mathbf{\Phi}(\tau,0)$$

  - STM matrix has identical symplectic property as known for linear dynamics about libration points in CRTBP
  - Asymptotically stable solutions cannot exist

$$\mathbf{J}\mathbf{A}^T = -\mathbf{A}\mathbf{J} \qquad \mathbf{\Phi}\mathbf{J}\mathbf{\Phi}^T = \mathbf{J} \qquad \mathbf{J} = \left[\begin{array}{c|c} \mathbf{0} & \mathbf{I} \\ \hline -\mathbf{I} & \mathbf{G} \end{array}\right]$$

1. $\det\left(\mathbf{\Phi}\right) = |\mathbf{\Phi}| = 1$
2. At least one Floquet multiplier has modulus of unity: $|\sigma_i| = 1$
3. The σ_i appear in reciprocal pairs (i.e. if σ_i is eigenvalue, then so is $\sigma_j = 1/\sigma_i$)

Periodic Coulomb Formations

Periodic Coulomb Formations

- Assume positions as simple harmonic oscillators
  - Solve time-varying charges that produce assumed motion
  - Motions are restricted (cannot be assumed arbitrarily)

- Relative instability measured via Floquet multipliers

- Dynamical coupling permits 3 solution families
  - Orbit-Normal oscillations only (1D)
  - Reference orbit-plane (2D) motions
  - Full state (3D) motions

Reference Orbit Plane Solutions

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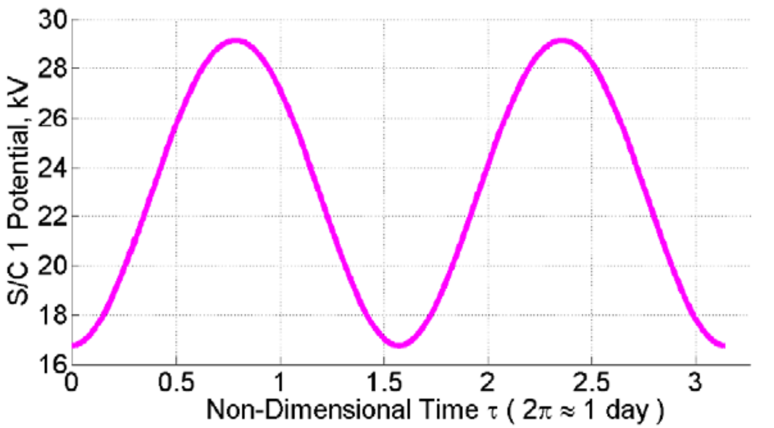
$$x(\tau) = A_x \cos(\theta_x \tau) \qquad y(\tau) = A_y \sin(\theta_y \tau)$$

1) $x(\tau)y''(\tau) - y(\tau)x''(\tau) = 0$ → $\theta_x = \theta_y$

2) Dynamical Coupling (Coulomb terms): → $\left(\frac{A_y}{A_x}\right) = \frac{-3 \pm \sqrt{9 + 16\theta^2}}{4\theta}$

- Two roots of quadratic yield two families (cases A/B)
  - Relative orbit is ellipse
  - Major axis along either radial (case A) or transverse (case B)
  - Short period (case B) orbits → least unstable

$$\tilde{Q}(r(\tau)) = \frac{-1}{\Psi(r)} \left[\theta^2 + 3 + \left(\frac{-3 \pm \sqrt{9 + 16\theta^2}}{2} \right) \right]$$



(b) Case B: $A_x < A_y$

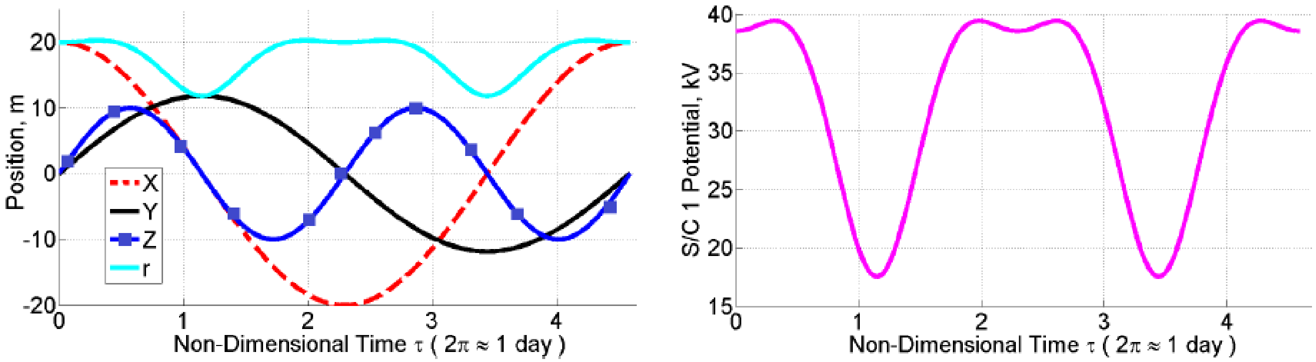
Full State Solutions

- Same in-plane assumed functions (same restrictions)
- Additionally $z(\tau) = A_z \sin(B_z \theta \tau)$

1) Out of plane frequency factor must be even integer → $B_z = 2, 4, 8 \ldots$

2) Time period no longer free, must satisfy: → $8\theta^2 + \left(-3 \pm \sqrt{9 + 16\theta^2}\right)\left[\theta^2(1 - B_z^2) + 1\right] = 0$

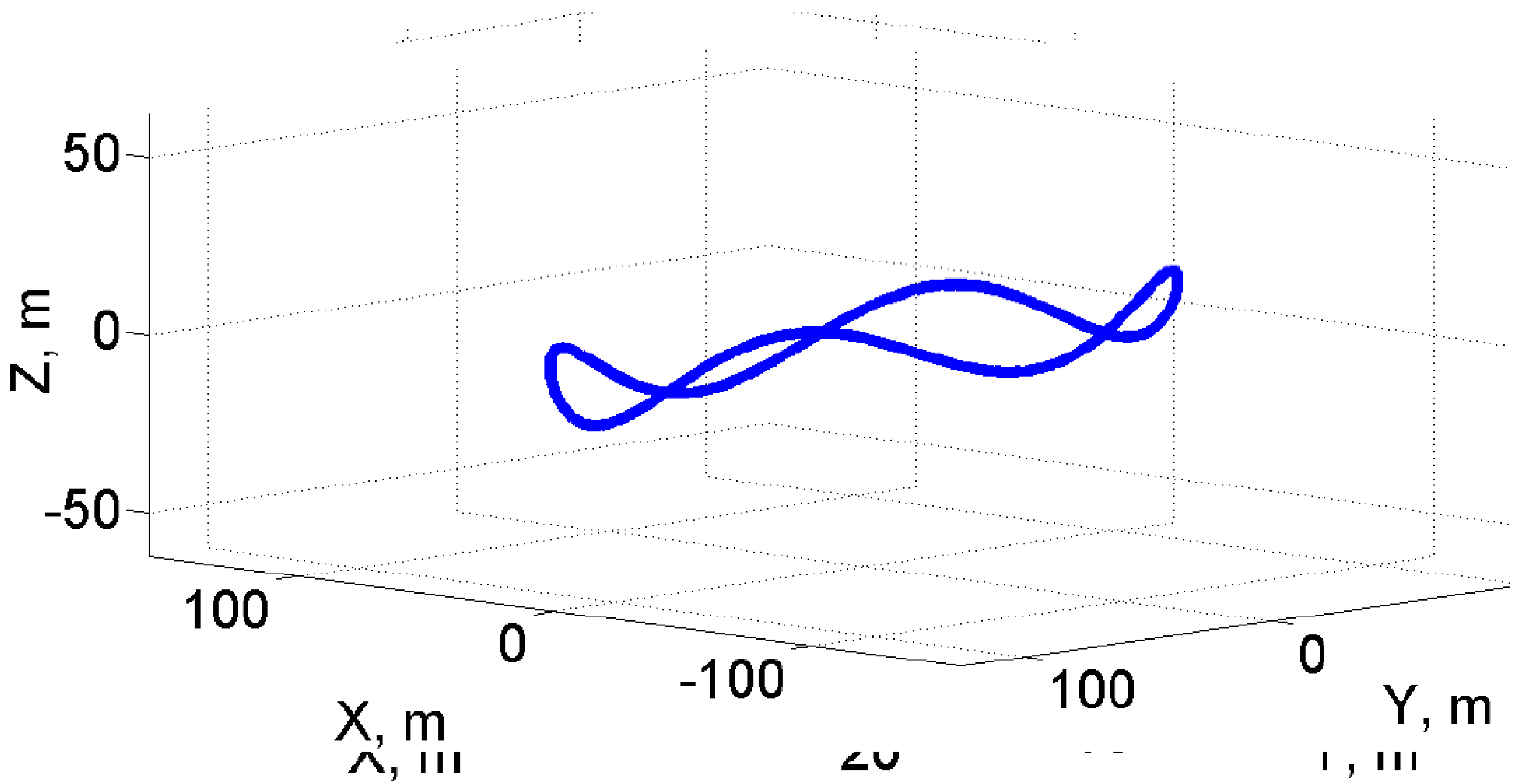
  – Projected trajectory is ellipse aligned as in case A/B
  – Case B orbits have smaller max. Floquet multiplier



(a) S/C 1 Position History (b) S/C 1 Potential History

Periodic Coulomb Formations

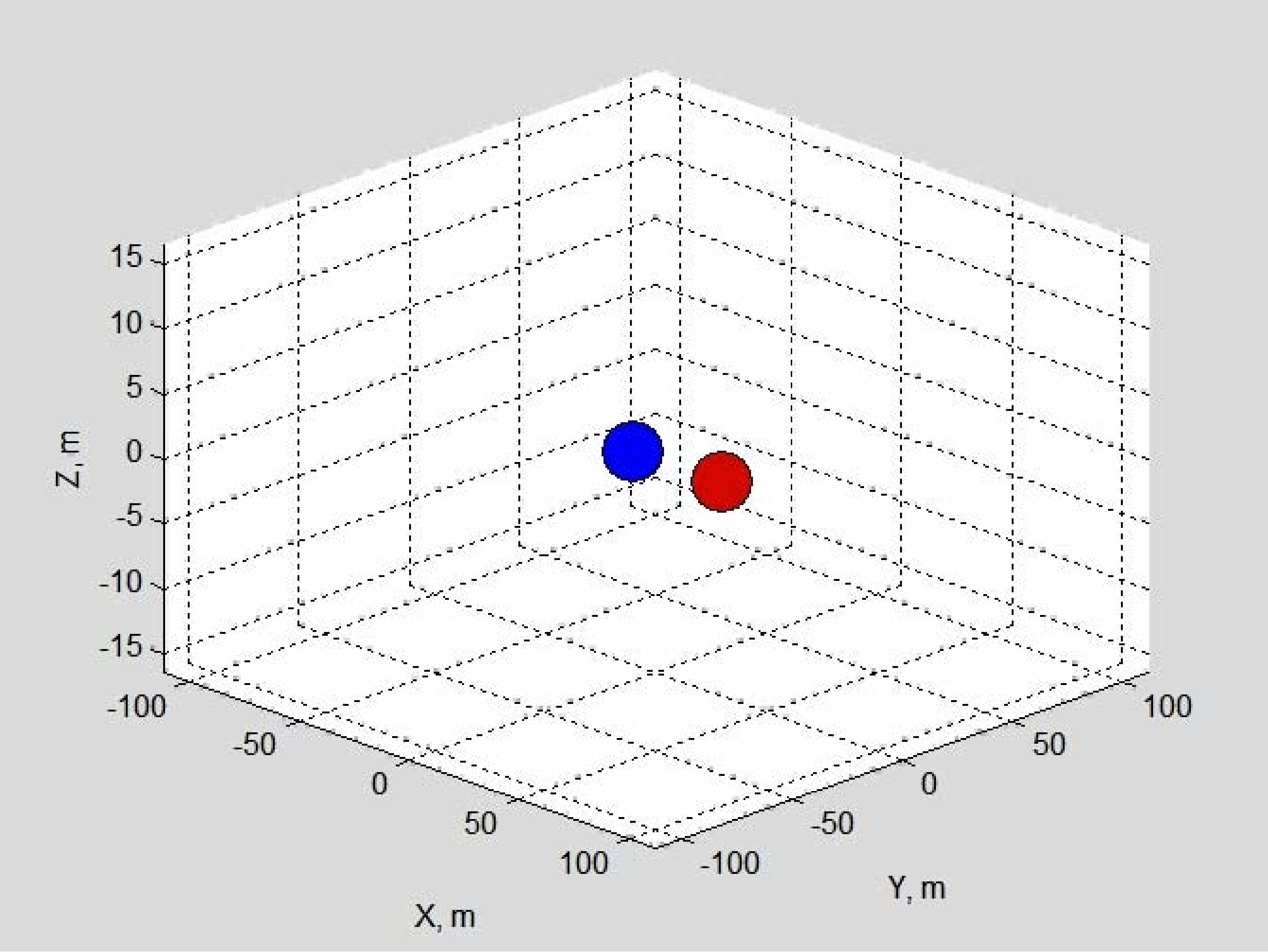
- Example relative orbit
  - S/C 1 Trajectory → like saddle, x-y projection is ellipse



Full State Periodic Solution: Case B, B_z=2

Circumnavigating 2-s/c Orbit

- Another Example
  - Both craft trajectories shown, different mass vehicles



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Perturbed Periodic Coulomb Formation Solutions Propagated in Inertial Frame

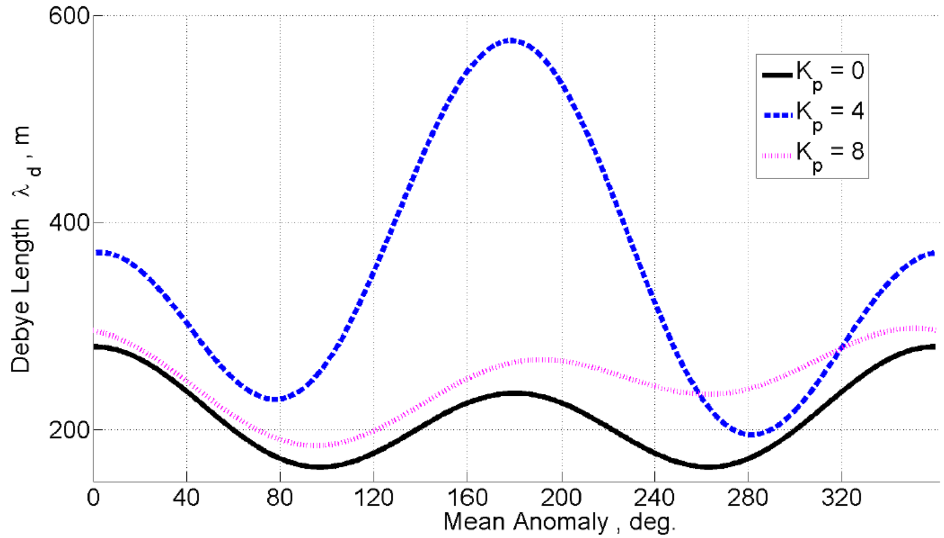
Perturbed Dynamical Model

- Accuracy of solutions in higher fidelity model
  - Revert back to Newtonian gravity, inertial frame
  - Transformations between ECI and Hill frames
  - Include primary perturbations

Solar Radiation Pressure using "Cannonball Model"

$$f_{\text{srp}} = C_R \frac{\pi R_{sc}^2\ \Theta}{m_i\ c}$$

Induced Perturbation due to Parametric Uncertainty (Model Error) in Debye Length



Perturbed Periodic Solutions

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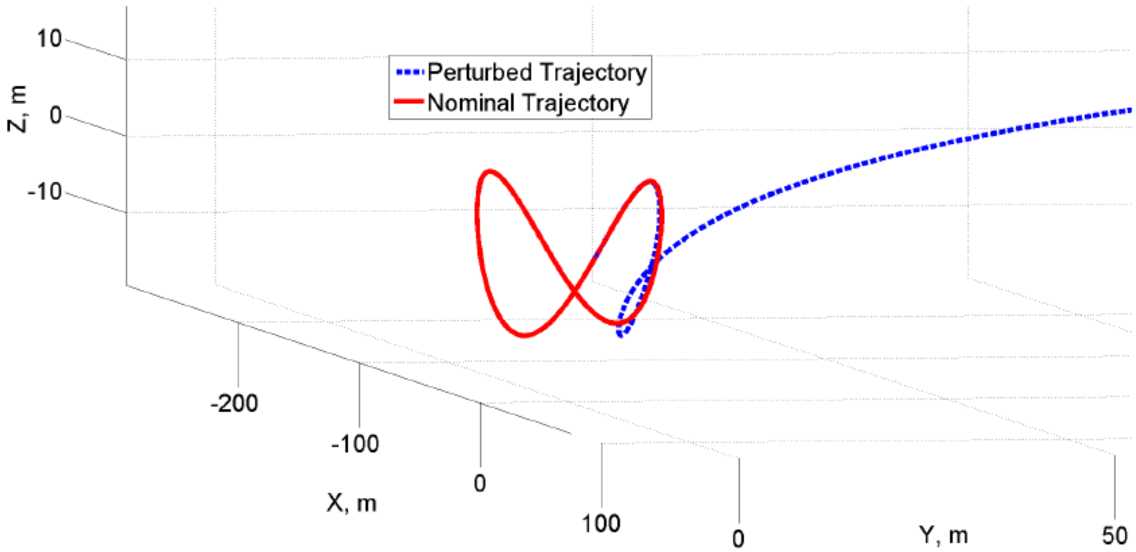
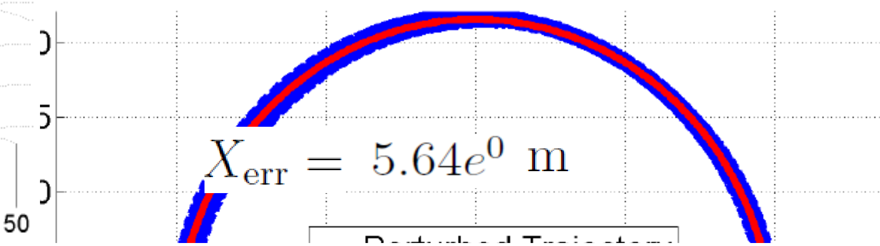
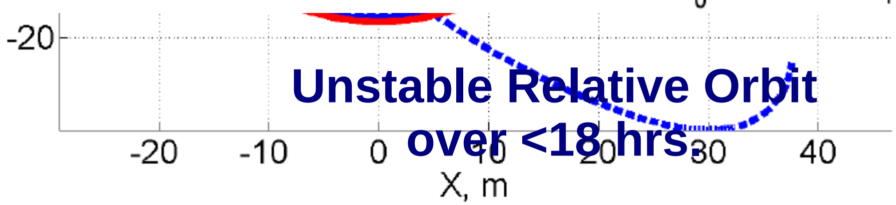


$$X_{\mathrm{err}} = 5.75e^{-2}\ \mathrm{m}$$

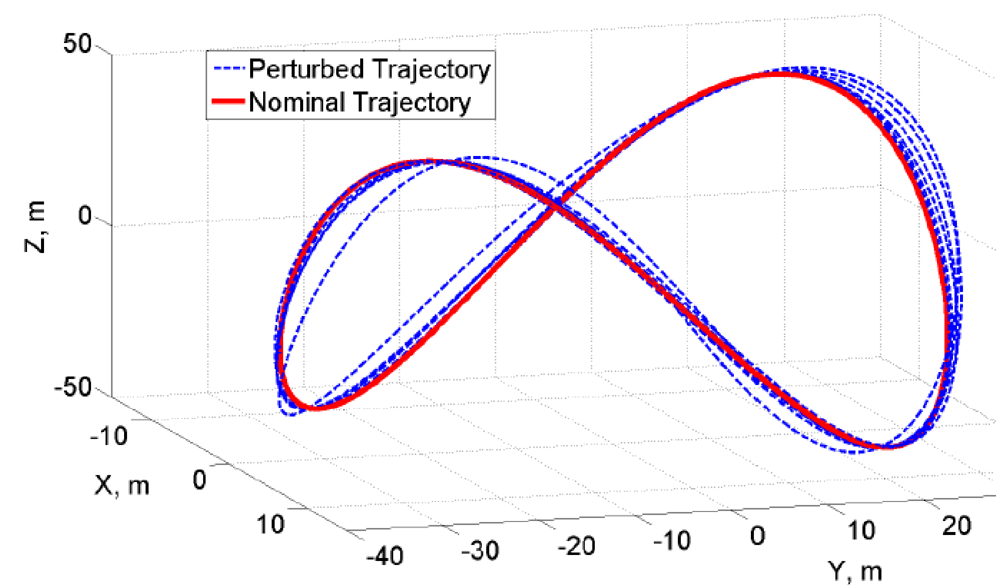


$$X_{\mathrm{err}} = 5.64e^{0}\ \mathrm{m}$$



Unstable Relative Orbit over <18 hrs.

Unstable Relative Orbit over 10 Revolutions



Stability over 40 Revolutions

Relative Orbit Exhibiting Marginal Stability over ~300 hrs.

Conclusions

- Application of new theory opens multiple pathways
  - The existence of periodic solutions enabled by open-loop electrostatic forcing
  - Stability properties assessed via Floquet theory
  - Some periodic solutions are very nearly stable
- Future Work
  - Other admissible periodic solutions (represent w/ finite Fourier)
  - Feedback control of periodic solutions
  - Test these solutions (and derive new ones) with relaxed assumptions and higher order Coulomb force modeling
  - Precisely measure, control, and estimate s/c potentials (and Coulomb forces) in order to realize these motions in practice?

QUESTIONS

Please refer further questions to Drew Jones: drjones604@gmail.com

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EXTRA SLIDES

Electrostatic Model Assumptions

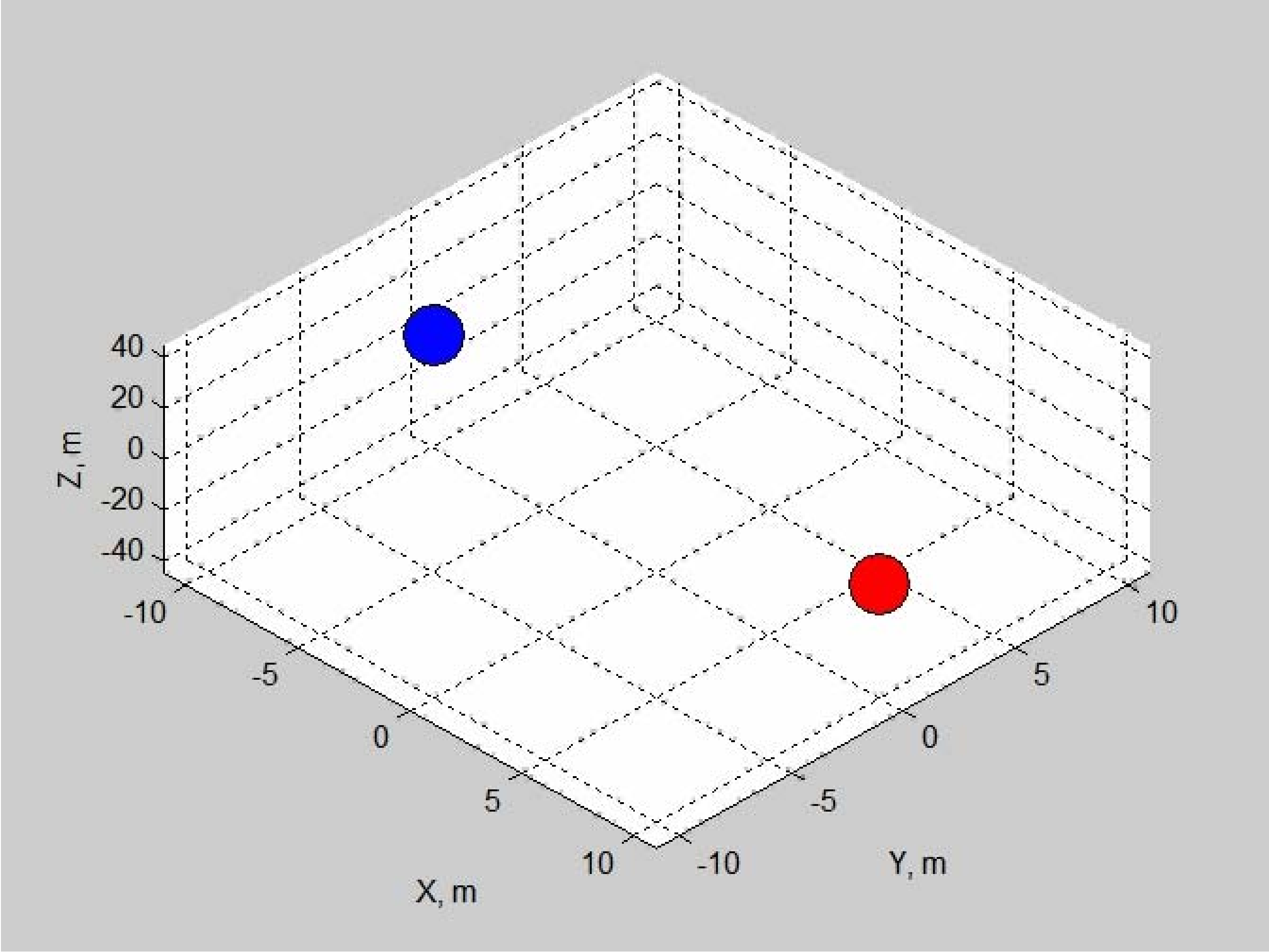
- True force between finite s/c: solution to time-dependent Vlaslov-Poisson PDE

$$F \approx k_c \frac{q_1 q_2 (1 + r_{12}/\lambda_d)}{r_{12}^2 \exp[r_{12}/\lambda_d]} \qquad \square_i = k_c \frac{q_i}{R_{sc}}$$

- Assumptions for approximate model accuracy:
  - Craft act equivalently as spheres at a distance
  - Conservative plasma shielding
  - Decoupled capacitances
  - Constant and finite λ_d (at GEO order of 100 m)
  - Formations near GEO (λ_d negligible over R_{sc})
  - Lower bound on separation distance: $r_{12} > 10\ R_{sc}$

Periodic Coulomb Formation

- Case A, $B_z = 2$, $A_x = 10$ m, $A_y = 6$ m, $A_z = 40$ m (17 hrs)



Periodic Coulomb Formation

- Case B, $B_z = 4$, $A_x = 20$ m, $A_y = 125$ m, $A_z = 5$ m (4 days)