Comparison between analytical and optimal control techniques in the differential drag based rendez-vous

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Differential drag, an attractive technique for LEO

ORBCOMM
40 kg
720 km

JC2Sat
18 kg
700 km

QARMAN
4 kg
350 km
Which approach fits best a realistic scenario?

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<th>VS</th>
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Outline

Differential drag in the QB50 constellation

Analytical & optimal control approaches

Comparison
1. Differential drag in the QB50 constellation

\[ \Delta F_{\text{drag}} = F_{\text{drag}, C} - F_{\text{drag}, T} \]
2. Analytical techniques

Assume:

- Uniform atmosphere
- $J_2$ perturbation
- Bang-bang
- Circular orbits

2. Proposed optimal control approach
2. Realistic estimation of the drag force

![Graph showing observed and fitted mean semi-major axis over time. The graph compares the observed values with those obtained using NRLMSISE-00 and Jacchia 71 models, including stochastic variations.](image)

- Time: 0 to 2 orbits
- Drag: 3e-6 N/kg to 9e-6 N/kg
- Mean semi-major axis: 6727 km to 6729 km
2. The planner: accuracy vs CPU burden tradeoff

Objective function

\[ t_f + W \int_0^{t_f} u^2 \, dt \]

Physical constraints

\[ \omega \in [\omega_{\text{min}}, \omega_{\text{max}}], \quad |u| \leq u_{\text{max}} \quad \forall t \in [0, t_f] \]

Boundary conditions

\[ X(0) = X_0, \quad X(t_f) = 0 \]

Dynamics

\[ \frac{d}{dt} X = f(X, t, \delta), \quad \frac{d^2}{dt^2} \delta = g(\delta, t, u), \quad \frac{d}{dt} \omega = h(\delta, t, u) \quad \forall t \in [0, t_f] \]

\[ x = \begin{cases} x_{\text{mean}} \\ y_{\text{mean}} \\ x_{\text{oscil}} \\ y_{\text{oscil}} \end{cases} \]
2. Numerical resolution via GPOPS

Optimal control ➤ Optimization problem

- ✓ No propagation
- ✓ No costates
- ✗ Large dimension problem

Initial guess: analytical solution
3. Optimal control greatly minimizes ADCS needs

\[ \sqrt{\frac{1}{t_f} \int_0^{t_f} u^2 \, dt} = 3e - 6 \, Nm \]

Analytical method

\[ \sqrt{\frac{1}{t_f} \int_0^{t_f} u^2 \, dt} = 1e - 8 \, Nm \]

Optimal control
3. Overshot increases maneuvering time

Optimal control

\[ t_f = 50 \text{ h } 17 \text{ m} \]

Analytical method

\[ t_f = 58 \text{ h } 42 \text{ min} \]
Conclusion

Future developments:

- Indirect optimal control
- Hybrid controller to account for computational cost peaks
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